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$$z_n = abc \div \left\{ \frac{1}{2\sqrt{3}} [(7+4\sqrt{3})^n - (7-4\sqrt{3})^n]\sqrt{abc(a+b+c)} + \tfrac{1}{6}[(7+4\sqrt{3})^n + (7-4\sqrt{3})^n - 2](ac+bc) + \tfrac{1}{6}[(7+4\sqrt{3})^n + (7-4\sqrt{3})^n + 4)ab \right\}.$$

NOTE ON THE BINOMIAL THEOREM.

BY GAETANO LANZA, ASSITANT PROF. OF MATH. AND MECH., MASS. INSTITUTE OF TECNOLOGY.

MR. EDITOR:—Allow me to call your attention to a demonstration of the Binomial Formula for positive integral exponents, given by Bobillier, which, though very simple and lucid, is not very generally known.

It does not involve at all the subjects of Permutations and Combinations, and hence in order to understand the Binomial Theorem the student is not obliged previously to master a number of propositions on the above subjects.

It would therefore seem very desirable that this demonstration should find a place in our text books and be taught in our schools.

For the benefit of any of your readers who may not be familiar with the notation employed, I will state that $\underline{|n}$ (=*factorial n*) is an abbreviated way of writing $1.2.3.4 \ldots\ldots\ldots n$, that is the product of the natural numbers from 1 to n inclusive; thus $\underline{|3} = 1.2.3 = 6$; $\underline{|4} = 1.2.3.4 = 24$, &c.

The demonstration is as follows:

The following three equations will be evident on inspection, viz.:

$$\frac{(x+a)^1}{\underline{|1}} = \frac{x}{\underline{|1}} + \frac{a}{\underline{|1}},$$

$$\frac{(x+a)^2}{\underline{|2}} = \frac{x^2}{\underline{|2}} + \frac{x}{\underline{|1}} \cdot \frac{a}{\underline{|1}} + \frac{a^2}{\underline{|2}},$$

$$\frac{(x+a)^3}{\underline{|3}} = \frac{x^3}{\underline{|3}} + \frac{x^2}{\underline{|2}} \cdot \frac{a}{\underline{|1}} + \frac{x}{\underline{|1}} \cdot \frac{a^2}{\underline{|2}} + \frac{a^3}{\underline{|3}}.$$

It is now proposed first to find an expression equal to

$$\frac{(x+a)^n}{\underline{|n}}$$

where n is any positive integer. In the above three results the following laws held:

I. The number of terms on the right hand side is greater by one than the exponent of $(x + a)$ on the left hand.

II. The exponent of x in the first term is the same as the exponent of the power to which $(x + a)$ is raised, and in each succeeding term the exponent of x is less by unity than that in the preceding term; while that of a follows the same law beginning at the other end of the series.

III. Each power of x or of a has for denominator its exponent taken factorially.

We shall now prove that these laws hold whatever positive integral value be assigned to n.

Suppose the laws to hold when the exponent is $n - 1$, that is suppose

$$\frac{(x+a)^{n-1}}{\underline{|n-1}} = \frac{x^{n-1}}{\underline{|n-1}} + \frac{x^{n-2}}{\underline{|n-2}}\cdot\frac{a}{\underline{|1}} + \frac{x^{n-3}}{\underline{|n-3}}\,\frac{a^2}{\underline{|2}} + \cdots\cdots$$

$$\frac{x^{n-r-1}}{\underline{|n-r-1}}\cdot\frac{a^r}{\underline{|r}} + \cdots\cdot + \frac{a^{n-1}}{\underline{|n-1}}.$$

Now multiply this identical equation by the following also identical, member by member, viz.:

$$\frac{x+a}{n} = \frac{x}{n} + \frac{a}{n},$$

and as the result of the multiplication we shall obtain the following:

$$\frac{(x+a)^{n}}{\underline{|n}} = \frac{x^{n}}{\underline{|n}} + \frac{x^{n-1}}{\underline{|n-1}}\cdot\frac{a}{\underline{|1}} + \frac{x^{n-2}}{\underline{|n-2}}\,\frac{a^2}{\underline{|2}} + \cdots\cdots\cdots$$

$$+ \frac{x^{n-r}}{\underline{|n-r}}\cdot\frac{a^r}{\underline{|r}} + \cdots\cdots + \frac{a^{n}}{\underline{|n}} \quad (1).$$

Hence, if the above laws hold for an exponent $n - 1$, they hold for the exponent n, which is greater than the former by unity; but they have been proved to hold for the exponent 3, hence they hold for 4, hence for 5, and so on, ad infiinitum; therefore they are true in the case of any positive integral exponent.

Now multiply both members of equation (1) by $\underline{|n}$ and we obtain

$$(x+a)^n = x^n + nx^{n-1}a + \frac{n(n-1)}{1.2}\,x^{n-2}a^2 + \cdots\cdots\cdots$$

$$+ \frac{\underline{|n}}{\underline{|n-r}\;\underline{|r}}\,x^{n-r}a^r + \cdots\cdots a^n,$$

which is the Binomial Formula.